

A dynamic, probabilistic model for city building

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Introduction

Many important aspects of interdependence between housing and transportation in cities have been dealt with in the literature of urban economics.¹⁻¹⁷ The model presented in this paper will focus on the element of uncertainty over time in city building. Exogenously given stochastic changes in the employment possibilities occurring over time in already built-up cities are explicitly considered. In order to adapt a city to such uncertain changes different alternative policy measures can be chosen. The model presented here is intended to be a step towards a tool that will make explicit economic evaluations of alternative master city plans possible.

In the first section some important characteristics of city building dealt with in the model are discussed briefly. Next an outline of the model is given, and the main assumptions used are presented. Then a more elaborated presentation of the main features of the model is given in the following section.

The possible course of development of the future working population within an upper and lower bound is simulated by means of a Monte Carlo technique. The technique used for these simulations is presented. The individual is assumed to choose modes and routes for commuting in an already built-up city that minimize his or her commuting costs and concepts used in this context are presented. The theory used to determine how many residences will be built when and where and in what kind of housing (low- or high-rise buildings) is presented and the optimization procedure is discussed.

The costs for different planning alternatives are calculated and the items included are presented. The objective function chosen is to minimize total costs for a given planning period. The mean value, as well as the variance from the mean value, are explicitly considered in the objective function.

Provisional, illustrative, numerical results are presented and some of the problems of implementation are discussed. In conclusion, the main features of the model are summarized and an outline of the authors' further research is given.

Characteristics of city building

Motives for city planning have been discussed in general terms in previous studies within the framework of welfare economics.^{1,3}

Here we wish to focus on some important phenomena in city building that will be explicitly dealt with in this paper, namely the interdependence between housing and transportation, the durability and irreversibility of many urban structures and uncertainty about the future.

The problems of building residences, working places and transportation systems in a city are strongly interrelated, since a solution chosen for one of these problems will greatly influence the solutions to the others, which may subsequently appear desirable.

In order to consider such mutual dependencies, it is necessary to compare one set of solutions to the above mentioned problems with another set of solutions. The following two extreme cases may exemplify the idea: either, mostly multi-storey housing and commuting mainly by public transport to a single business centre; or sprawling residential areas with single-family houses and commuting mainly by car to several business centres at different locations.

Will the objectives of the city best be satisfied by one of these two planning extremes or another alternative? To determine this it is necessary to compare the different total alternatives in terms of costs and benefits. (However, in this paper such a comprehensive study is not presented.)

In a previous study, comparisons were made considering explicitly the interdependence among population density, city shape and transportation system in a static, deterministic model for new towns.² However, the implications of some important characteristics of city building were not dealt with there. Such characteristics include the durability of many city structures and the limited *ex post* substitutability due to irreversible production processes.

If a residential house has been built, then the number of storeys (limited substitutability), location (limited mobility) and physical lifetime (indivisibility over time or durability) of that house are given entities. Of course, it is possible to transform the *ex post* situation to an *ex ante* situation by demolishing and building new urban structures or converting (within certain limits) some urban structures directly. This means that irreversibility and durability are

This paper represents a summary of a part of the authors' book to be published in the near future. In this book the model will be described more thoroughly.

no longer constraints in city building. But this will involve extra costs that depend heavily on what has been built.

The fact that urban structures are very durable and highly irreversible means that an existing pattern of urban structures in a built-up city will strongly influence city building for a long time.

In the dynamic approach all future exogenous changes in the city must be envisaged at the initial decision point in time. Economic evaluation of one master city plan should consider not only the city area as a whole but also all the changes expected to occur during the total time span covered by the plan. But uncertainty concerning many of the key variables is an additional problem, since deviations from the simulated development will imply extra costs. As a general rule urban structures require a production period. It is therefore necessary to make the decision to supply such products in advance of the appearance of the demand, according to the efficiency condition that supply has to equal demand. If, for instance, the supply of housing should fall short of demand, due to the appearance of an unexpectedly large working population, this would represent a welfare loss and extra resources would be needed to produce the necessary housing. This can perhaps be done relatively smoothly if such possible deviations from an expected path are considered in the land reservation policy. Otherwise the extra costs might be substantial.

If on the other hand the supply of residences, offices and roads should turn out to be too great, significant losses will have been incurred due to the above mentioned facts that investments in building these facilities are highly durable and irreversible. The residences, offices and roads that will not be used represent erroneous investments, in this case.

Outline of model

Aim

The aim of the model is to try to evaluate the economic consequences of given temporal planning alternatives for a city with given initial structure which is to be adapted to changes in the working population.

Data

To solve this problem it will be assumed that the following variables are known: (1) At the initial point of time t_0 the existing city structure is geographically completely determined. This means that the city area will be considered to be a set of nodes (city nodes) where the coordinates (x, y) , type of land use (residence, working places, open space, parking place, road nodes, etc.), net area to be built and intensity of land use (number of storeys) at each city node are known.

Also the transportation layout (configuration of the road network) and the possible modes of commuting are defined from every residential node to the working places, assumed to be concentrated in a central area called Central Business District (CBD). The possible modes of commuting considered are walking, driving and parking a car and bus commuting.

(2) Economic data: These known variables are related to building and running costs connected with housing and transportation. Lifetime, building* and maintenance

costs for residences, offices (working places), parking places, roads, etc. are given. Costs for converting from one type of facility to another are included. Velocities for each mode of commuting are known.† For vehicles, depreciation, maintenance, petrol costs, etc. are also given. Values for commuting time and space demanded for every facility are known. All these data are assumed to be constants. (This restriction in data can be easily relaxed in this model because of numerical procedures used.)

(3) Expected working population limits: In this set of data, it is assumed that upper and lower limits for all possible values of the working population are known.

(4) Long run planning decisions: These decisions concerning physical planning are also taken at the initial point in time, but they may be put into effect at any time during the period of interest for the study. Decisions of this type are mainly concerned with land reservations (where and how much) and future changes in the initial transportation layout.

Results

From these known variables the following types of results can be obtained with the model:

A number of simulated possible developments of the working population (by means of a Monte Carlo technique).

For each simulated working population path the set of planning decisions (complementary decisions to the long run planning decisions given as data*) is obtained, namely: number of apartments; locations of residences; number of storeys; modes of commuting at different locations and city shape.

For each working population path the cost consequences of the plan are evaluated.

These costs occurring at each point in time include the following items: building and maintenance costs of residences; working places; parking places; roads, etc. as well as costs of vehicles (depreciation, maintenance, petrol, oil, etc.) and commuting time.

Revision costs due to possible deviation of the actual number of the working population from the simulated value at each point in time are also considered.

These costs may arise when the actual number of the working population is greater than the simulated one, owing to the increased demand for housing and commuting, and in the form of a penalty for welfare losses due to the shortages of apartments, etc. In the opposite case, when the actual number of the working population is smaller than assumed, savings in commuting costs occur, but the investments already taken in housing and commuting have been excessive. The present values of all these costs for the time of the study are summed.

Evaluation of objective function. The types of results presented immediately above can be summed, through the

† For the car and bus modes of commuting only the coefficients v_0, v_1 are given, where the velocity v is expressed by the formula $v = v_0 + v_1 f$, and f is the unknown variable for congestion. f = number of vehicles per lane and per unit of time. This value depends on the length of the peak hours assumed (Trush).

* In this paper a plan will be defined as the union of these two types of decisions: (a) long run decisions taken at initial point of time and known as data and (b) complementary decisions obtained as results in the model.

* The data presented for these costs are the coefficients a_i, b_i, c_i of the regression line (parabolic curve) with building cost of the item i as a function of the number of storeys.

simulated paths and expressed in probabilistic terms, usually in mean values and standard deviations.

The objective function of the model is expressed as the sum of the mean value and some fraction of the variance of the total costs. The evaluation of this objective function is also obtained as a result.

By successively changing the given long-run planning decisions the 'best' plan (i.e. the one that minimizes the objective function) can be obtained by an iterative use of this model.

Computational steps

In order to obtain the results presented above from the given data the following computational steps are taken:

(a) Simulation of a working population path $WP\eta(t)$ for the time plan of study $(0, T)$. A probability value $\pi(\eta)$ is associated to this path, by means of a Monte Carlo technique.

(b) Simulation of the commuting in the city at each time intersection t_a to be studied ($a = 1, 2, \dots, NT$).

Every individual living at node n will choose the mode and route of transportation that minimizes his individual commuting costs. This individual commuting will create congestion (mainly negative external effects). In order to determine the commuting pattern an iterative procedure is used, i.e. the congestion is represented by starting values which will be changed in successive steps of the iteration until a convergence in the values of the congestion between two consecutive steps is reached.

(c) Determination of the population density distribution at time t_a . The differences in the population due to the changes in the working population will be allocated by use of the following exponential gross density function.*

$$D(a, n) = A_a e^{-b_a c_b(a, n)}$$

where A_a and b_a are parameters that vary with time.† $c_b(a, n)$ indicates the individual commuting costs at node n , just obtained from the previous step. The formula given above is applied only to a node n , when new residences can be built there. If there are already buildings at n , then at the current time t_a , the formula should be replaced by one of the following:

$$D(a, n) = D(a', n) \quad a' < a$$

(if the inhabitants remain)

or:

$$D(a, n) = 0 \quad a' < a$$

(if the inhabitants leave, i.e. if there are vacant residences)

depending on the working population path (expansion or contraction of the working population).

(d) Supply of residences: Assume a constant value of the demanded habitable space per person and let it be α , then two cases may occur. If n is a node where new residences can be built at time t_a , then the number of storeys at node

n , $\alpha_1(a, n)$, is given by the expression:

$$\alpha_1(a, n) = \frac{\alpha}{\lambda(n)} A_a e^{-b_a c_b(a, n)} \leq \alpha_{1\min}$$

where $\lambda(n)$ is the given ratio between the built-up area and the total area at node n . The restriction of $\alpha_1(a, n) \geq \alpha_{1\min}$ (a minimum number of storeys) is due to the data assumptions used. More precisely the cost of building storeys is assumed to be quadratic function of the number of storeys α_1 , and so reaches an absolute minimum value for $\alpha_1 = \alpha_{1\min}$.

If n is a node where residences already have been built at t'_a previous to the current time t_a , then:

$$\alpha_1(a, n) = \alpha_1(a', n)$$

due to the irreversibility and durability of residence buildings. (It is important to note that the computational steps b , c and d are closely related so that they must be repeated iteratively a number of times until convergence in all the values is obtained.)

(e) Economic evaluation of the consequences of the planning decisions at time t_a .

(f) When all the computational steps (b) to (e) have been taken for each point of time t_a ($a = 1, 2, \dots, NT$), i.e. NT times, then the total costs for the time $(0, t)$ are summed to present values.

(g) These computational steps from (a) to (f) are repeated to simulate a given sufficient number (NS) of working population paths.

(h) From the values obtained in the computational step g , the mean value of total costs and its standard deviation, through the NS simulated working population paths can be calculated and the objective function evaluated in this way.

According to these results comparisons between different sets of given long-run decisions can be effectuated or, alternatively, changes in the given set of long-run decisions can be introduced in order to ameliorate the plan.

The application of this model, even to simple cases, demands the use of a digital computer, because the large amount of mathematical operations needed to reach the results are impossible by hand computation. This situation is accentuated when real city configurations have to be studied.

Main assumptions

In assumptions to the main assumptions already presented the following have been used in the model:

Evaluation criterion: minimize total costs. Only housing and commuting costs are explicitly calculated. A complete cost-benefit analysis cannot be carried out using the model at this stage. But, nevertheless, in the model some costs usually not considered in a normal cost minimization study have been included. These costs or savings are mainly related with commuting time.

The possibility of different evaluations of time for different commuting modes has been introduced. The influence of congestion on time costs has been taken into account in this evaluation criterion.

A limitation of the model is that only a commuting transportation system is considered. In reality the comparative advantages of commuting modes differ between

* This formula is a generalization of the well-known Colin-Clark formula $D = A e^{-br}$. By replacing the distance r by the value of c_b in the formula the city commuting facilities will then in this way be implicitly considered.

† The values of A_a are determined by an equilibrium condition, namely, the densities summed over the whole city area must equal the total population. The values of b_a are obtained from the minimization of the objective function.

different uses. Thus public transport has its comparative advantages for commuting at peak hours. Cars will play a greater role in a transportation system designed for all kinds of travelling in a city than in a system designed for commuting only. A more extensive discussion of the limitations of the objective function used will be given later in the paper.

The working population changes over time in a way that is outside of the control of the city authorities. The city structure has to be adapted to such exogenously given changes. Even if the city is not an isolated one, the interdependencies between cities are not studied here.

Exogenous changes in the working population are stochastic within an upper and lower bound over the planning period. The upper and lower bounds are subjectively determined by the city planners. The possible outcomes for each point of time are assumed to follow a given probability density function. (See section on simulation of future working population.)

All places of work are concentrated in the city centre (monocentric cities). All the employees have to commute to and from the CBD every working day. This assumption is usual in the literature of urban economics, but it is a very restrictive assumption, because it is not possible to deal with problems of real growing cities, i.e. to investigate the consequences of different number of business centres, their locations and sizes. In reality, only a fraction of the employees in a city goes to the CBD and the rest of them commute to some sub-centre.

There are no differences in tastes and information between households. Of course, there exist differences in people's preferences and it may turn out to be efficient for the society to physically separate such groups due to the existence of economies of scale. (Segregation patterns due to differences in income are not discussed under this point.)

residence space is price and income inelastic.

and that in reality the *per capita* demand for residence space varies with both price and income. Then the demand for housing space will vary for instance with the location in a city as the land rent (apartment rent) varies.

Households are indifferent between living in a single-family house and in an apartment of the same size in a multi-family house. It is usually possible to have immediate access to a private garden when a single-family house is chosen, while a household in a multi-storey house usually has to rely on public open space such as parks. Such substitutes are considered inferior by many households. Of course, it is difficult to determine how large an area of public park assigned to a household of a high-rise building apartment would be required to offset the utility of a private garden of a given size and quality attached to a single-family house.

Average per capita income is constant over time; Income distribution is completely uniform. If different income classes are assumed together with a zoning in the city area according to differences in environmental benefits, it would be possible to simulate segregation patterns of reality assuming some ranking principle.

It is obvious that all these general assumptions have been chosen for nothing but simplicity. They ought to be gradually removed by future studies in order to approach a more realistic city planning situation.

Simulation of future working population

Let us assume that at a given point of time t_a , the working population level is $WP(t_a)$. At that point of time the development of the future working population $WP(t)$, between the two successive points of time, t_a and t_{a+1} , is supposed to lie between the two limits WP_{\min} and WP_{\max} (Figure 2):

$$WP_{\min}(t, t_a) \leq WP(t) \leq WP_{\max}(t, t_a)$$

where:

$$t_a \leq t \leq t_{a+1}$$

It can be assumed for simplicity that the limits of changes in the working population, i.e. $WP(t) - WP(t_a)$, are functions of the time lag only $\tau = t - t_a$ (hypothesis of time homogeneity).^{*} Then the following expressions hold:

$$WP(t) \leq WP(t_a) + \Delta WP(\tau)$$

$$WP(t) \geq WP(t_a) - \Delta WP_{\min}(\tau)$$

$$t_{a+1} \geq t \geq t_a$$

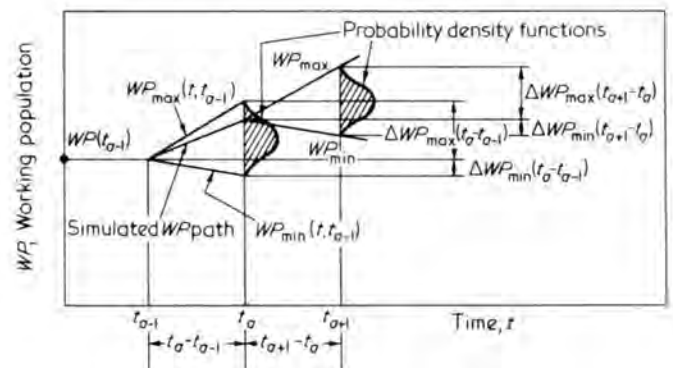


Figure 1

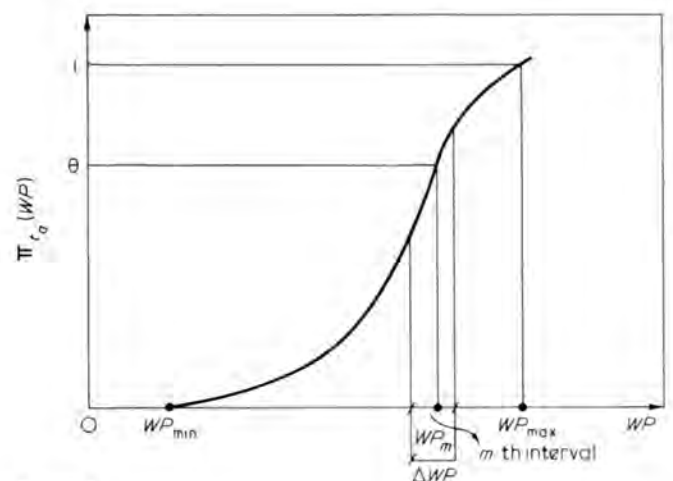


Figure 2

^{*} This approach was suggested independently both by Alf Carling, Department of Economics, University of Stockholm and Folke Snickars, Department of Mathematics, Royal Technical High School of Stockholm.

The functions $\Delta W_{\max}(\tau)$ and $\Delta W_{\min}(\tau)$ are given as data in the model and they are assumed for convenience to be linear. It is important to note that these expressions are only valid between the two consecutive points of time t_a and t_{a+1} .

The functions of the time lag $\tau = t - t_a$ only, $\Delta W_{\max}(\tau)$ and $\Delta W_{\min}(\tau)$, are given as data in the model. The occurrence of the forecasted value of $WP(t_a)$ at a time t_a is associated with a probability distribution function $\Pi_{t_a}[WP(t_a)]$ (Figure 2), i.e. $\Pi_{t_a}[WP(t_a)]$ = the probability that the working population at time t_a is smaller than $WP(t_a)$.

Obviously, $\Pi_{t_a}[WP_{\max}] = 1$ and $\Pi_{t_a}[WP_{\min}] = 0$.

For numerical reasons it is convenient to divide at each time t_a the total interval (WP_{\min}, WP_{\max}) of this probability distribution function into M small parts of equal length ΔWP_a . For these divisions, let us use the index m ($m = 1, 2, \dots, M$) there exists a probability $\pi_m \cdot \Delta WP_a$ that WP will fall within the m th division. The value of this probability can be obtained as the difference between the two ordinates of the probability distribution function corresponding to the extreme values of this m th part. The middle point of the m th interval is WP_m .

Similarly, the working population path going from the initial point at time t_0 to the end of the planning period, $t_{NT} = T$, can be described as a random process called η . This process will be represented by the set of values of the working population at some particular point of time t_a ($a = 1, 2, \dots, NT$), i.e.:

$$\eta \equiv [WP(t_1), WP(t_2), \dots, WP(t_a), \dots, WP(t_{NT})] \quad (1)$$

It is possible to associate a probability density function $\pi(\eta)$ with the occurrence of η .

$\pi(\eta) \Delta WP_1 \cdot \Delta WP_2 \cdot \dots \cdot \Delta WP_{NT}$ = probability that some working population path η^* , lies between the following two working population paths, η and $\eta + d\eta$, where:

$$\begin{aligned} \eta &\equiv [WP(t_1), WP(t_2), \dots, WP(t_{NT})] \\ \eta + d\eta &\equiv [WP(t_1) + \Delta WP_1, WP(t_2) \\ &\quad + \Delta WP_2, \dots, WP(t_{NT}) + \Delta WP_{NT}] \end{aligned}$$

assuming $\Delta WP_1, \Delta WP_2, \dots, \Delta WP_{NT}$ are sufficiently small increments of the working population.

In order to simulate a working population path η for the planning period, a Monte Carlo technique will be used. At time t_a it is assumed that the working population is WP_a with a given probability $\bar{\pi}_{t_a}$, where:

$$WP_a = WP_m \quad \bar{\pi}_{t_a} = \pi_m$$

The m interval is defined by the following condition:

$$\pi_{t_a} \left[WP_m - \frac{\Delta WP_a}{2} \right] \leq \theta \leq \pi_{t_a} \left[WP_m + \frac{\Delta WP_a}{2} \right]$$

θ is a random number, between (0, 1) with a rectangular probability density function. It can be generated by a digital computer (pseudo-random number).

Then the probability of occurrence of a given working population path η , assuming a Markov process* is:

$$\pi(\eta) = \bar{\pi}_{t_1} \cdot \bar{\pi}_{t_2} \cdot \dots \cdot \bar{\pi}_{NT} \quad (2)$$

where:

$$\eta \equiv [WP_1, WP_2, \dots, WP_{NT}]$$

* A Markov process is used only for simplicity. More general random processes could be used in this simulation technique.

Individual's choice of mode and route of commuting

At each intersection t_a every employee in the CBD living at node n will choose some mode of commuting, j , among walking, commuting by car or by bus ($j = 1, 2, 3$) and also a route $r = r(a, n)$ and an entry node $ne = ne(a, n)$ to this route.

The rule is to choose the values of j , $ne(a, n)$ and $r(a, n)$ that minimize the individual's commuting costs.

Let $c_{b_1}(a, n)$ be the minimum individual commuting cost by walking from the node n to the CBD. Similarly, $c_{b_2}(a, n)$ is the minimum individual commuting cost from the node n to the CBD considering all the possible car routes r and all entry nodes, ne , to each route.

Analogously, $c_{b_3}(a, n)$ is the minimum individual commuting cost from the node n to the CBD taken into consideration every possible bus route r and all the corresponding bus stop nodes ne . Now it is possible to define for each residential node n , the individual commuting cost $c_b(a, n)$ as the minimum among the three above defined values of $c_{b_j}(a, n)$.

Mathematically, this can be expressed as follows:

$$\begin{aligned} c_b(a, n) &= \min_j c_{b_j}(a, n) \\ &= \min_{j, ne, r} [c_{b_1}(a, n), c_{b_2}(a, n), c_{b_3}(a, n)] \quad (3) \end{aligned}$$

The minimal value of j determines the individual's choice of commuting mode. The corresponding route $r(a, n)$ and entry node of this route that minimize the particular $c_{b_j}(a, n)$ determine the individual's route of commuting.

General expressions for these individual commuting costs are given below.

(a) Walking ($j = 1$)

$$c_{b_1} = p_{t_1} \cdot t_{b_1}$$

where: t_{b_1} = walking time required going at a constant speed from a residential node to the CBD-centre; p_{t_1} = time value of walking.

(b) Car commuting ($j = 2$)

$$c_{b_2} = \min_{ne, r} \left[t_{b_2} \cdot p_{t_2} + \frac{1}{n_c} \cdot x_2(d_2, g_2, pp_2) \right]$$

where: t_{b_2} = amount of time for a car commuting trip per commuter. In the computation of this value t_{b_2} the changes in the congestion along the commuting trips has been explicitly taken into account; p_{t_2} = time value of car commuting per commuter; n_c = number of persons per car; and x_2 = pecuniary costs of car commuting per trip and per car. It is written as a function of the commuting distance, variable congestion along the commuting-trip and parking fees at the CBD. The parking fees have been calculated from the apartment rents per unit of ground area at the CBD (obtained endogenously in the model).

(c) Bus commuting ($j = 3$)

$$c_{b_3} = \min_{ne, r} [t_{b_3}^{(1)} \cdot p_{t_1} + t_{b_3}^{(2)} \cdot p_{t_3} + x_3]$$

where: $t_{b_3}^{(1)}$ = time used in walking to and from the bus stop and waiting time; $t_{b_3}^{(2)}$ = time used sitting in the bus; p_{t_3} = time value of bus commuting per commuter and x_3 = bus fare (endogenously determined from marginal cost pricing).

The details of the computation of the values of $c_{b_1}(a, n)$, $c_{b_2}(a, n)$ and $c_{b_3}(a, n)$ are given elsewhere.¹⁸

Supply of residences

Usually in the literature of urban economics the gross density D^* at a node n is assumed to be an exponential function of the distance $r(n)$ to the CBD centre, i.e.:

$$D^*(n) = A^* e^{-b^* r(n)}$$

where A^* and b^* are two parameters.

In this study the above formula is replaced by:

$$D^*(n) = A e^{-b \cdot c_b(n)}$$

where: $c_b(n)$ is the individual commuting costs at node n and has been already defined.⁴ A and b are parameters.

The geometrical configuration of a transportation system and the choice of commuting mode will, in a city, influence the shape of a city as well as the population density. The density will usually decrease with the distance from a radial main road, but it will also continue to decrease along the local roads. And if the commuting transportation system of a city is mainly based on public transport, it may be advantageous to concentrate the residences in the areas around stops and stations for buses and trains. This dependence is considered here using the expression presented above with the density as a function of the individual's commuting costs.

In order to determine the density distribution two interrelated aspects should be considered: the relative densities between different locations in a city and the absolute level of the population density. The relative density will be determined not only on the basis of individual commuting costs but also from a minimizing criterion based on some objective function. The absolute level of the population density will be obtained by an equilibrium condition: the total population should equal the sum of the densities over the whole city area. This means that the two parameters A and b will be found in this study by means of the following two conditions:*

(a) Equilibrium condition, i.e. the total population is equal to the sum of the population living at all of the residential nodes, n . Total population = $\sum D(n) \cdot an(n)$; n = residential node; $an(n)$ is the built-up area at node n ; $D(n) = D^*(n)/\lambda(n)$ = net density.

(b) Minimizing the objective function OF presented later in the paper (section on illustrative results). That means that the values for A and b have to be chosen in such a way that the minimum of OF will be reached at the same time as the above equilibrium equation is satisfied.

Once the density $D(n)$ at each particular city location is known, it is possible to derive the number of storeys $\alpha_1(n)$ needed in the housing at this location, according to the following considerations.

Let α be the habitable area demand per person (exogenously given). Then the number of storeys, α_1 , at this

location should equal the density times this value of α :

$$\alpha_1(n) = D(n) \cdot \alpha = \alpha \cdot A e^{-b \cdot c_b(n)}$$

or the dynamic case:

$$\alpha_1(a, n) = \alpha D(a, n) = \alpha \cdot A_a e^{-b_a \cdot c_b(a, n)} \quad (4)$$

But some limitations to equation (4) have to be introduced due to the following efficiency condition:

The average building costs per habitable unit area is a function of the number of storeys as shown in Figure 3. Then it is apparent that it is not efficient to build residences with fewer storeys than $\alpha_{1\min}$, if both housing and transportation costs are included in the objective function as is the case in this study. In fact, let $\bar{\alpha}_1$ and α_1^* be a pair of numbers of storeys for which building costs per habitable unit are equal, i.e. $p(\bar{\alpha}_1) = p(\alpha_1^*)$. Then, of the two possibilities of building with this particular construction cost, $\bar{\alpha}_1$ ($\bar{\alpha}_1 > \alpha_1^*$) produces a lower value of the objective function OF than α_1^* due to the fact that the transportation costs have been reduced by diminishing the average commuting distance. For that reason, if in the solution of equation (4):*

$$\alpha_1(a, n) < \alpha_{1\min}$$

then $\alpha_1(a, n) = 0$, and the area at node n is reserved for later use.

Optimization procedure

Before starting the description of the calculation of total costs it may be interesting to discuss in broad terms optimization procedures that might be used in the model and those actually chosen.

A vector notation will be introduced for the working population path ($WP_1, WP_2, \dots, WP_{NT}$), density distribution parameters (b_1, b_2, \dots, b_{NT}), and (A_1, A_2, \dots, A_{NT}), namely:

$$\left. \begin{aligned} WP &= \{WP_a\} \\ b &= \{b_a\} \\ A &= \{A_a\} \end{aligned} \right\} \text{vector dim. } (NT \times 1)$$

$TC = \text{total costs}$

The density distribution at time t_a is determined for the unimproved area by the expression:

$$D_a(n) = A_a e^{-b_a \cdot c_b(n)}$$

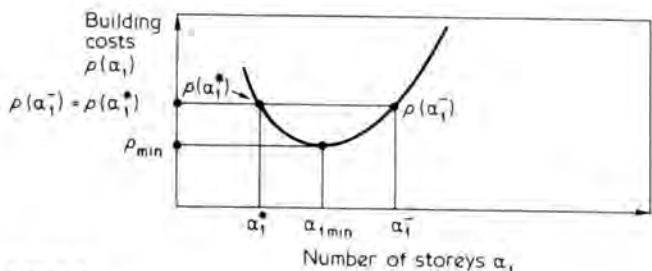


Figure 3

* Use of the number of storeys $\alpha_1(n)$ instead of the conventional density concept $D(n)$, has the advantage of making it possible to visualize a city in three-dimensional space with houses of a different number of storeys.

* The density $D(n)$ ought to be written as a function of the individual commuting costs $c_b(n)$. The chosen exponential function can be given a behavioural representation. Rather than assuming this exponential form it is possible to let the functional form of the density distribution be determined in the optimization procedure. But this simple exponential form is probably sufficient to illustrate some of the fundamental characteristics of possible population distributions.

The variable TC is a functional of WP , b , and A , i.e.:

$$TC = f(WP, b, A) \quad (8)$$

An equilibrium relationship exists at each point of time t_a among WP , b and A , namely the total population is equal to the sum of the density over the whole city area. This equilibrium condition can be set up as follows:

$$A = \phi(WP, b) \quad (9)$$

The dynamic problem can be stated in the following terms: Given WP , find the only independent vector b (the vector A is obtained from (9)) that minimizes TC . A possible and natural procedure to find this minimal value of b could be to use dynamic programming.

An additional aspect is introduced in the model, namely the uncertainty in the WP . Mathematically that implies WP is a random vector represented by $\{WP^{(\eta)}\}$. In order to tackle this problem two possibilities are described below:

(1) For a sample $WP^{(\eta)} \in \{WP^{(\eta)}\}$ the procedure just described to solve the dynamic problem can be used, namely, given $WP^{(\eta)}$, b is found by minimizing TC .

In reality b is a random vector, because it depends on $WP^{(\eta)}$ and so can be called $b^{(\eta)}$. Also the minimum total costs obtained for this minimal $b^{(\eta)}$ is a random variable called $TC^{(\eta)}$.

If a simulation procedure is used, the distributions of $b^{(\eta)}$ (also $A^{(\eta)}$) and $TC^{(\eta)}$ can be obtained by solving the dynamic problem for each $WP^{(\eta)}$ simulated repetitively.

The values of b chosen as the optimal complementary solution may for example be the mean value of $b^{(\eta)}$, i.e. \bar{b} . The total costs may be described by the distribution of $TC^{(\eta)}$. This method of defining and obtaining the optimal b is quite cumbersome and so was not used in this model.

(2) A constant value for the vector b is provisionally assumed independent of the sample vector $WP^{(\eta)}$. Then, A is found from equation (9), i.e.:

$$A^{(\eta)} = \phi(WP^{(\eta)}, b)$$

That means $A^{(\eta)}$ is a random variable, but not b .

The total costs obtained from the values of $WP^{(\eta)}$ and b – they are not the minimum ones – can be represented by the random variable $TC^{(\eta)}$ where $TC^{(\eta)} = f(WP^{(\eta)}, b, A^{(\eta)})$.

The following objective function OF (deterministic variable), related to the random variable $TC^{(\eta)}$, is considered in the model:

$$OF = \overline{TC^{(\eta)}} + (\overline{TC^{(\eta)}} - \overline{TC^{(\eta)}})^2 \mu_3$$

where μ_3 is some factor of risk aversion. That means, by simulating different samples $WP^{(\eta)}$, the value of OF can be found for a given vector b . Summarizing the above discussion: For a given vector b , a value of OF can be calculated.

Thus the problem now has been reduced to that of finding a vector, b , such that OF is minimum. If the number of points of time considered, NT , is small, the value of b can be found from an enumerative technique (assuming some discrete values for b). In other cases, a standard nonlinear optimization programming can be used, in particular, the steepest gradient search method.

Calculation of total costs

Introductory remarks

In order to compare current costs (such as time and petrol costs) with costs that occur only once over a period

of many years (e.g. housing and road building costs), it is necessary to assume an economic lifetime NL and an interest rate ir . Aggregation of the different cost items can then be calculated either as present values or costs *per annum*. In this study present values have been chosen.

The calculation of the total costs at time $t(a) = t_a$ is divided into four parts:

- (1) costs related to simulated changes in nodes
- (2) costs related to simulated changes in links
- (3) costs related to simulated running costs (vehicle, oil and tyres, and commuting time)
- (4) costs related to simulated deviations from a simulated path

Costs related to changes in nodes and links

The following building costs per unit of built-up area have been considered: residences p_0 , working places p_1 , parking places p_2 , bus stops p_3 , car way intersections p_4 , bus stop + car intersections p_5 , open space p_6 , 'other activities' $p_7 = 0$, transitional nodes $p_8 = 0$, roads p_9 and sidewalks p_{10} .*

Costs related to links are for roads and sidewalks. The costs of a particular link i will be denoted as TC_i where:

$$TC_i = C_1^* + C_2^*$$

C_1^* is related to the cost due to increasing the number of lanes l in the positive direction of the link and, similarly, C_2^* in the negative direction, i.e. directions back and forth to the CBD.

Running costs (vehicles, oil, tyres and commuting time)

The costs of running vehicles can be divided into costs for cars C_c and costs for buses C_b . The cost of commuting time is denoted by C_t .

These commuting costs will be evaluated through the period from $t(a)$ to $t(a+1)$, at present value, i.e. $t = 0$.

Then we obtain for the running costs TRC :

$$TRC = (C_c + C_b + C_t) \frac{-(1+ir)^{-t(a+1)} + (1+ir)^{-t(a)}}{ir}$$

Revision costs

These costs are due to the deviations from the simulated working population path and they should be taken into account at each revision point of time $t_a = t(a)$. If $WP_a = WP(t_a)$ is the working population at time t_a , two possible types of deviations exist: expansive and contraction deviations.

Expansive deviations. In this case the actual working population WP is larger than the expected one WP_a .

An average value of the working population in this case is:

$$WP^e(t_a) = \frac{1}{\pi^e(t_a)} \int_{WP_a}^{WP_{\max}} WP \cdot \pi_{t_a}(WP) dWP$$

where:

$$\pi_{t_a}(WP) = \frac{d}{dWP} \Pi_{t_a}(WP)$$

* p_0, p_1 and p_2 are written as functions of the number of storeys:

$$p_i(\alpha_i) = a_i \alpha_i + b_i \alpha_i + c_i$$

where a_i, b_i and c_i are known coefficients.

$\Pi_{t_a}(WP)$ is the probability distribution function defined earlier in this paper, and:

$$\pi^e(t_a) = \Pi_{t_a}(WP_{\max}) - \Pi_{t_a}(WP_a)$$

is the probability of the occurrence of an expansion deviation.

For this value of the working population $WP^e(t_a)$, there are three types of costs:

- (a) Costs for converting the residences nearest to the CBD into offices.
- (b) Costs building residences for the extra population $WP^e(t_a)$ and people displaced due to the above mentioned conversion plus the welfare loss because of a shortage in housing.
- (c) Incremental transportation costs for the city. The evaluation of each of these cost items will be given below.

Conversion costs. The set of residential nodes N_1 , where conversion from residences to offices will take place, is obtained from the equation:

$$\sum_{n \in N_1} \frac{\alpha_1(n)}{\beta_e} \lambda(n) \Delta x \Delta y = WP^e(t_a) - WP_a$$

where: N_1 is the set of nodes nearest to the CBD and such that $c_b(a, n)$ is minimum and β_e is the space demanded per employee.

Residential building costs and welfare losses. The allocation of the extra population $WP^e(t_a) - WP_a$ and of the displaced population given by the expression:

$$\sum_{N_1} \frac{\alpha_1(n)}{\alpha} \lambda(n) \Delta(n) \Delta x \Delta y$$

or equivalently:

$$[WP^e(t_a) - WP_a] \frac{\beta_e}{\alpha}$$

would be obtained. Then the corresponding supply of residences is also given there.

A penalty for the welfare loss because of a possible shortage in housing must also be included as a cost. It has been included simply by multiplying the residential building costs by a factor ($= 2$).

Incremental transportation costs. These costs are obtained in the usual way for the new city assuming that the population there is $WP^e(t_a)$.

In order to keep the admissible traffic congestion down in the city for this new increased population $WP(t_a)$, it will be assumed that it is possible to increase Trush (length of the commuting period), if necessary.

The increment will be the difference between the two running costs due to the two levels of working population, i.e. the simulated one, WP_a , and the increased one, $WP^e(t_a)$.

Contraction deviations

In this case the actual level of the working population WP at time intersection t_a is smaller than the simulated one WP_a .

The average value of the working population will be in this case:

$$WP^c(t_a) = \frac{1}{\pi^c(t_a)} \int_{WP_{\min}}^{WP_a} WP \Pi_{t_a}(WP) dP$$

where the probability of the occurrence of a contracting case is:

$$\pi^c(t_a) = \Pi_{t_a}(WP_a) - \Pi_{t_a}(WP_{\min})$$

In the contraction case, the residences with larger individual commuting costs will be empty where N_i is the set of empty residences, in order to fulfil the equilibrium condition:

$$\sum_{n \in N_i} \frac{\alpha'_1}{\alpha} s(n) \Delta n(n) = WP_a - WP^c(t_a)$$

The savings in running costs will be computed as the difference between the transportation costs occurring for the two working population levels, namely WP_a and $WP^c(t_a)$.

Objective function

The objective function chosen is to minimize total costs due to all the decisions taken during the planning period T . Total costs $TC(t_a)$ at one point of time, t_a , can be defined as follows:

$$TC(t_a) = \sum_{N^*} TC_k + \sum_{L^*} TC_l + TRC + \mu_1 \cdot IC^+ \cdot \pi^e(t_a) + \mu_2 \cdot IC^- \cdot \pi^c(t_a) \quad (6)$$

where: N^* set of nodes with changes; L^* set of links with changes; TC_k , TC_l and TRC have already been defined. IC^+ are the costs due to the adaptation of the city to a greater working population than simulated (expansion). IC^- corresponds to the adaptation costs in the contraction case. The probabilities of the working populations, $\pi^e(t_a)$ and $\pi^c(t_a)$, have already been defined. μ_1 and μ_2 are factors reflecting a risk aversion. A neutral value for these factors may be unity.

Now if uncertainty is introduced for the whole planning period, the changes in working population have to be considered during this period. Then several paths of the working population will be simulated. The occurrence of one of these paths is associated with a probability value $\pi(\eta)$ calculated earlier in the paper.

If

$$u = \sum_{n=1}^{NT} TC(t_a)$$

represents the total costs associated with one simulated path during the whole planning period, the mean value is as follows:

$$\bar{u} = E(u) = \sum_{\eta=1}^{NS} \left[\sum_{a=1}^{NT} TC(t_a) \right] \frac{\pi(\eta)}{\sum_{\eta=1}^{NS} \pi(\eta)}$$

where: $TC(t_a)$ is the total costs at time t_a including the possible adaptation costs; N is the number of simulated paths and NT is the number of points of time.

This mean value is perhaps not sufficient in an evaluation of total costs to permit comparison of different plans. It may also be of interest to include the variance from this mean value in this evaluation.

A higher mean value of one plan may be preferred to a lower mean value of another, if the variance of the first one is sufficiently lower, since that implies less risk. Then

the expression for the objective function may be given as follows:

$$OF = E(u) + \mu_3 E[(u - \bar{u})^2] \quad (7)$$

where $\bar{u} = E(u)$ and μ_3 is a weighting factor.

Some limitations have to be kept in mind when interpreting the results of this model with the above presented criterion.

First: Only housing and commuting costs are explicitly considered. This means for instance that the transportation system chosen will be designed for commuting only, i.e. not for other personal trips neither for the transportation of goods. In addition not all cost items involved in commuting are included in the objective function, such as costs of traffic accidents, noise and fumes, etc.

Second: The evaluation criterion is restricted to a minimization of total costs. But such a criterion cannot be the ultimate formulation of the society's efficiency objective. It may be replaced by maximization of the following function:

$$\text{Net benefits} = \text{gross benefits} - \text{total costs}$$

Let us illustrate this important difference by a concrete example. According to a minimization of total costs, no single-family houses at all ought to be built.* But even if minimum total costs for single-family housing are greater than total costs for an alternative with multi-family housing, the net benefits of the former might, to some extent, be larger than the net benefits of the latter. (Features like access to a garden and a higher degree of privacy connected with single-family housing.) However, the strength of the developed model presented in this paper is that it can be used to calculate how large the extra benefits must be at the minimum in order to motivate single-family housing, from the efficiency point of view of the society.

Illustrative results

The ultimate aim of this modelling work is to obtain a tool which can facilitate evaluations of alternative master city plans. This is a goal in a long-range research programme. No alternative plans for real cities will be evaluated in this paper. Nevertheless, in order to illustrate the use of the present model, two hypothetical plans will be compared, with the risk that they will be considered somewhat artificial. The purpose is mainly to illustrate the importance of considering land reservation and design of the city structure in a long-range planning perspective, due to the characteristics of city building especially dealt with in this study (i.e. interdependence, durability and irreversibility combined with uncertainty over time).

First, a new town is supposed to be constructed for a working population of 150 000 (corresponding to 300 000 inhabitants). This new town is built during a period of five years and during this time it is assumed that there is complete certainty with regard to the development in working population.

* The cost minimum when only building costs for residences are considered is reached with multi-family dwellings of approximately three-storeys. Taking transportation costs in a city into consideration as well as in this model, the cost minimizing number of storeys will be even greater.

Two planning approaches are possible in this case. First, a longer planning period (20 years) is assumed. Naturally, uncertainty about the future is there from the initial point in time. However, in this simplified illustration no uncertainty is assumed to exist concerning the first five-year period. Uncertainty is supposed to exist only for the following 15-year period.

In the second approach the city may be designed only for the first period of five years (assuming complete certainty), and a static situation with no changes is expected for the time after this period. This approach can be a simulation of some type of 'day-to-day' planning method. In Figure 4 (a and b) a schematic representation of these two situations is presented.

In the first plan land reservations are made at the initial point of time, t_0 , for the whole planning period of 20 years. Thus land reservations are also explicitly considered for the period 5–20 years. Of course, there are many ways to make the land reservations and to lay out a transportation system. Here only the following land reservations will be investigated (see also Figure 5a): (a), around the CBD for

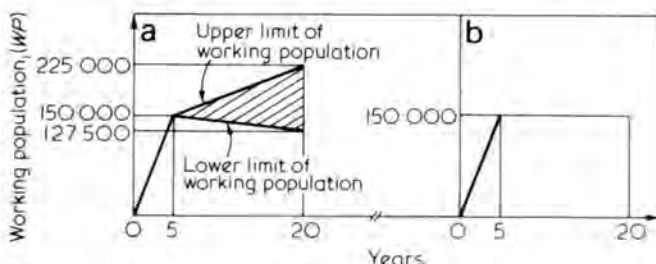


Figure 4 (a), plan I; (b), plan II

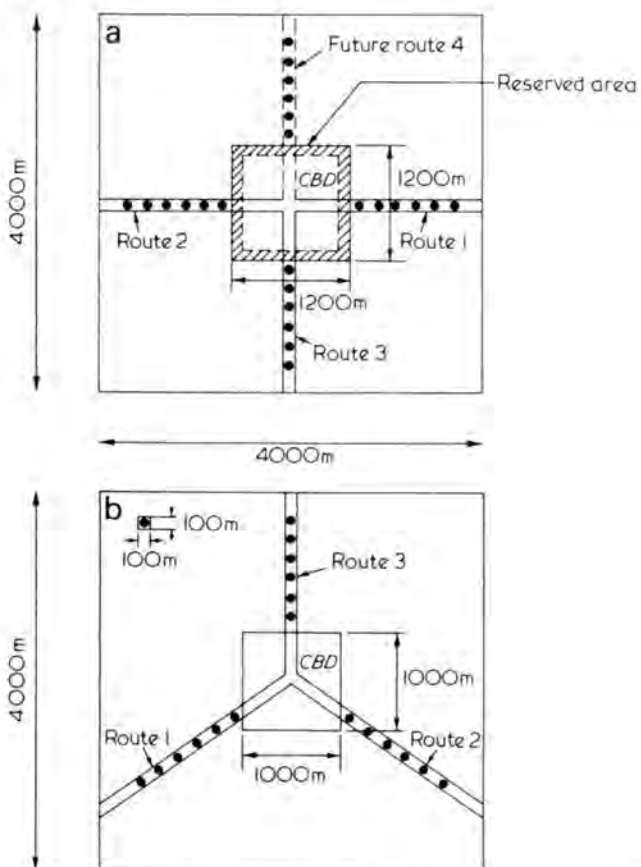


Figure 5 (a), plan I; (b), plan II. (■), city node; (●), transportation node (bus stop and/or car entry node)

an expansion of the working places, and (b), for an extra new road.

The 'day-to-day' plan case implies that the possible development of the working population after the first five-year period is not considered at all in the planning decisions made at the initial point in time.

Nevertheless, the city planner is obliged to realize, even in this case, that the city must at some point in time be adapted to the development of the working population after the first five years. For simplicity the assumptions and the method of dealing with the data are supposed to be the same as in the first case except for the fact that the adaptation in this case has to be made conditional on the city structure chosen without considering the possible development after the first five years (see *Figure 5b*).

The adaptation possibilities explicitly considered are the following: (a) Conversion of the residences nearest to *CBD* into a required number of working places. New residences have to be built in the outskirts of the city for the displaced inhabitants. (b) More congested traffic, and (c) Building new residences at the outskirts around the built-up city at an inefficiently large number of storeys (in comparison with the number chosen for the first plan).

By simulating the consequences of these two different ways of planning (long-run *versus* short-run) the resulting two plans can be compared.

To enable a comparison between these two plans several working populations should be simulated. Some results corresponding to one simulated path are presented here. Thus the results will only be of interest as illustrations of how the model works.

The time intersections considered are the years 0, 5, 11, 14 and 20. In *Figures 6–11* some computer drawings are presented for the years 5 and 20 respectively. In *Figures 6–8* results concerning the shape of the city, the commuting modes chosen, the individual commuting cost and the density distribution chosen for the year 5 according to plan I are presented. *Figure 6* shows that most of the commuters choose to go by bus and by walking, but a limited number of commuters chooses to go by car. *Figure 7* shows the noncircular isolines of the individual commuting

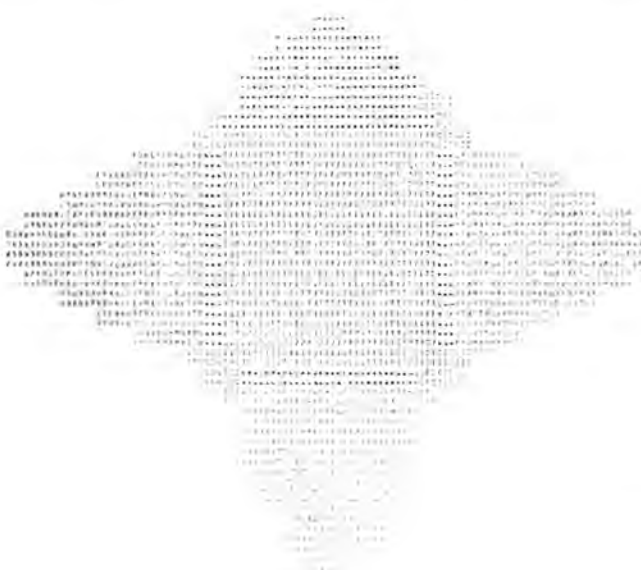


Figure 6 City shape and modes of commuting (plan I, year 5). W, walkers; B, bus commuters; C, car commuters, and 1, 2, 7, offices, parking places and unused areas

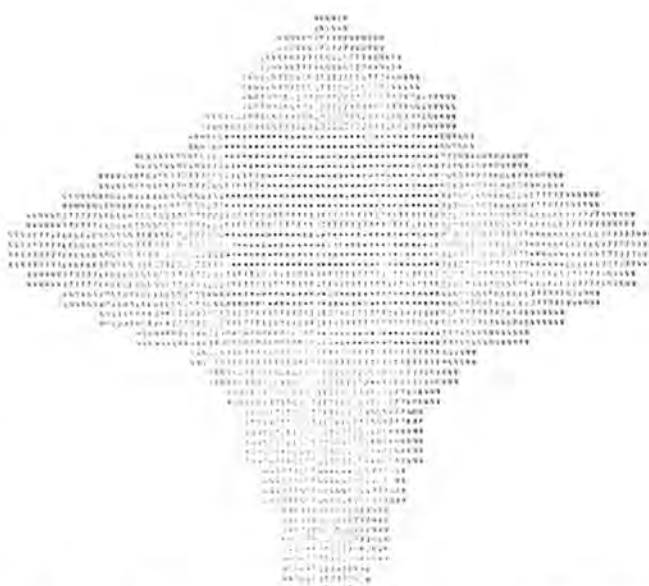


Figure 7 Representation of the individual commuting costs (plan I, year 5). 9, \$0.31 (highest value); 0, \$0.17 (lowest value); T, offices, and P, parking places



Figure 8 Population density distribution for households of bus commuters (plan I, year 5). 8, 0.159 inhabitants per m²; 0, 0.157 inhabitants per m²; T, offices, and P, parking places

costs. In *Figure 8* the corresponding population density distribution for commuters by bus is presented.

In *Figures 9 and 10* results according to plan I for the year 20 are presented. Comparing *Figures 6 and 9* it can be seen that commuting by bus has replaced commuting by walking along the new road. The corresponding development of the density distribution for bus commuters is illustrated in *Figure 10*. (Compare with *Figure 8*.)

In *Figure 11* results for the year 20 according to plan II are shown namely the city shape and the commuting modes chosen. The commuting takes place mainly by bus and by walking. Some car commuting occurs in addition to the commuting. It is also of interest to notice that the land reservations made for working places in the *CBD* are not sufficient at this point in time. Therefore, it is necessary

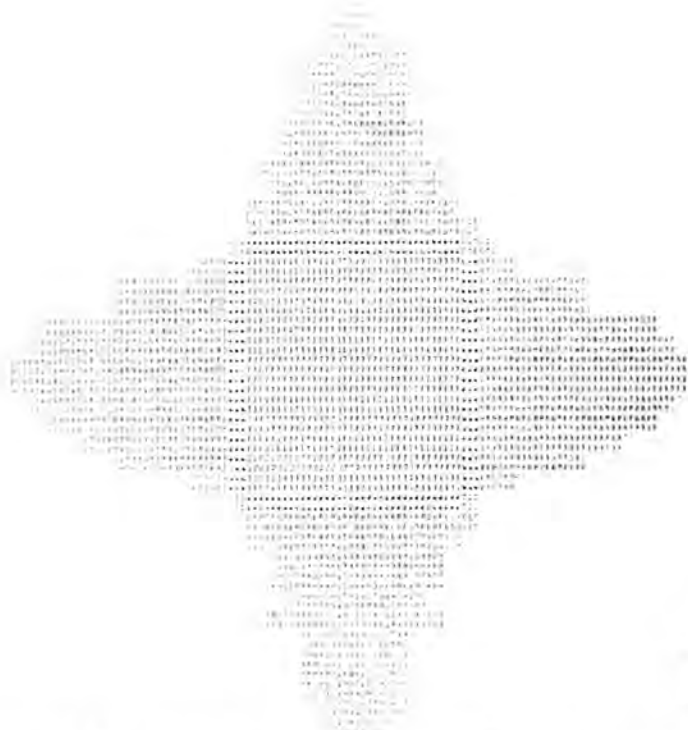


Figure 9 City shape and modes of commuting (plan I, year 20). W, walkers; B, bus commuters; C, car commuters. 1, 2, 7, Offices, parking places and reserved areas

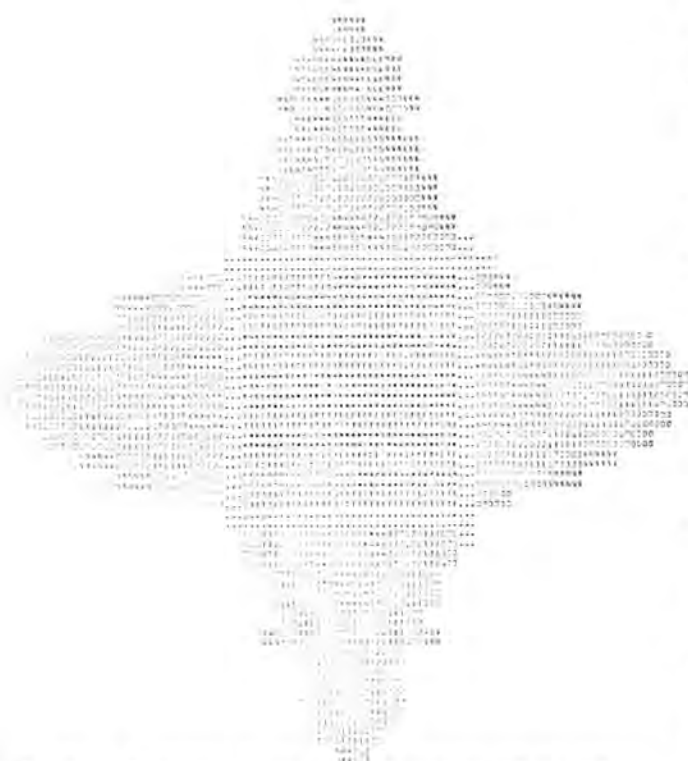


Figure 10 Population density distribution for households of bus commuters (plan I, year 20). 9, 0.161 inhabitants per m^2 ; 0, 0.157 inhabitants per m^2 ; T, offices, and P, parking places

to convert residences close to the planned CBD-area into offices. Then, new residences have to be built for the displaced people.

It would not make much sense to compare the total costs of the two plans when only one simulated path is

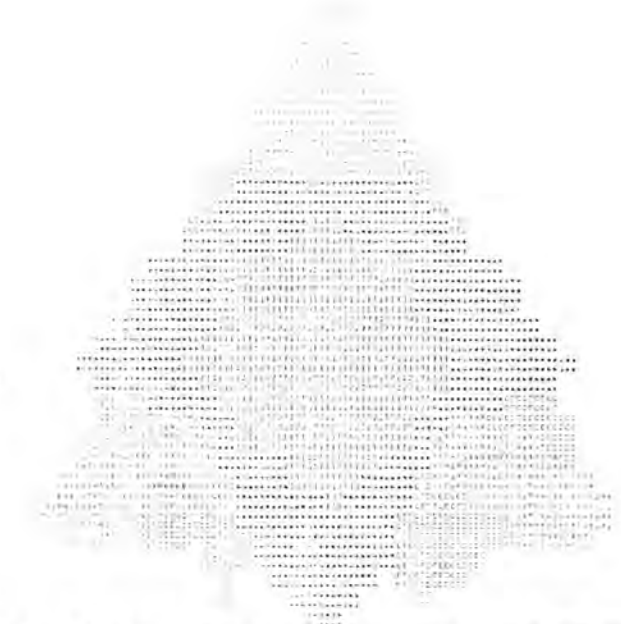


Figure 11 City shape and modes of commuting (plan II, year 20). W, walkers; B, bus commuters; C, car commuters, and 1, 2, offices and parking places

considered. However, as an illustration the variations in total costs for plan I are presented in Table 1.

Planning and implementation

If it is possible to find efficient master city plans, the 'next' step is to identify efficient means of implementation, so that the desired development can be realized. Such possible means may be taxes, charges, subsidies, public utility rates, laws, standards, permits to construct buildings, public investment, etc.

For instance, in Sweden the national policy in housing will strongly influence the production and consumption of housing. Production of new residences is substantially subsidized. This means that more, bigger, and qualitatively better apartments are built with than without such subsidization. Similarly several demand increasing stimuli exist to promote ownership of single-family houses.

The purpose here is to point out that a considerably greater consumption of residential space can result in the presence of the above-mentioned 'advantages' than in their absence.

The existing principles in use for different implementations means may function as important constraints on the choice set. Inferior alternatives may be chosen with existing pricing principles, financial constraints, laws, etc. than with alternative value constraints. It is therefore important to try to evaluate a first-best solution independent of such constraints. Let us illustrate this problem with the concrete example of housing.

Let us assume that a city is planned according to the distribution of demand for apartments between single-family houses and multi-family houses resulting from the existing set of taxes, pricing policy and institutional conditions. One simplifying assumption can be that the apartments are rented at equilibrium prices. The city's transportation system is built without consideration of the present special restrictions in pricing and financing principles, but only to constraints imposed by the availability of real

Table 1 Variations in total costs (plan I)

| Year | Costs along one path* | | | | Revision costs* | | | | | | Total |
|-------------|-----------------------|--------------------------|---------------|--------------------------|--|-------------|-------|--|-------------|-------|--------|
| | Working population | Costs connected to nodes | Running costs | Costs connected to links | Working population expansion deviation | Probability | Costs | Working population contraction deviation | Probability | Costs | |
| 5 | 150 000 | 1186.6 | 38.3 | 64.5 | — | — | — | — | — | — | 1289.4 |
| 8 | 161 197 | 63.1 | 20.0 | 15.6 | 162 465 | 0.034 | 14.7 | 154 656 | 0.441 | -0.9 | 99.7 |
| 11 | 166 935 | 30.8 | 17.7 | 0 | 170 022 | 0.201 | 26.6 | 163 508 | 0.244 | -0.4 | 53.8 |
| 14 | 178 717 | 46.2 | 16.3 | 0 | 179 790 | 0.024 | 10.3 | 171 747 | 0.456 | -0.7 | 62.4 |
| 20 | 196 042 | 79.5 | 28.0 | 0 | 200 267 | 0.094 | 24.6 | 186 258 | 0.360 | -1.3 | 109.4 |
| Total costs | | | | | | | | | | | 1614.7 |

* In millions of dollars.

resources. Traffic is to be charged for fumes, congestion, noise and risks of collisions in peak traffic. Similarly, charges on the use of oil with a high sulphur content in residential heating are to be imposed. The city designed according to such a plan may differ from present-day cities in that there will be fewer single-family houses, fewer apartments heated individually by oil or electricity, less automobile traffic, and correspondingly more multi-family houses, district heating and public transportation.

Thus a first-best solution ought to be evaluated and then the pricing principles, taxes, laws, principles to permit the construction of buildings, etc. changed in order to implement such a solution. However, if there are absolute constraints on the use of some means, it is important to consider such constraints already in the evaluation of alternatives constrained with respect to the implementation means used – second-best solutions – instead of the ‘unconstrained’ first-best solutions.

Also, it is important to review other existing institutional conditions. Division of the planning responsibility among different, more or less independent, housing, heating and transportation authorities may cause inefficiency. A coordinating central decision-making agency may be necessary in order to implement efficient solutions from the society's point of view.

Summary

The problem of interdependence between housing and commuting in a city has been analysed within the framework of welfare economics. Uncertain changes over time in the working population has been considered by means of a dynamic, probabilistic model. The characteristics of irreversibility and durability in city building have been explicitly dealt with. The ultimate objective is that the model after further development will be an auxiliary tool in city planning.

Some important features of the model are:

The development of the working population over time is handled as a random process.

A master city plan is divided into two groups of decisions: long-run decisions are exogenously given to the model and decisions that will be endogenously determined.

The objective function consists of the mean value of total costs and a fraction of the variance across all the simulated working population paths. These total costs, due to the consequences of a plan, include items for

housing and commuting. Different time evaluations are used for the different modes of commuting.

Residential building costs as a function of the number of storeys has been included.

Costs and savings due to differences between the simulated and actual working population development have been taken into account.

The total city area is divided into elementary areas (city nodes) for different city facilities: residences, working places, parking places, roads, reserved areas for future use, etc. That means, for instance, the Central Business District (CBD) area where the total working places exists, is endogenously determined.

Minimum individual commuting cost is the criterion for the individual's choice of mode and route of commuting.

Several modes of commuting and layouts of the transportation system can be studied with the model.

The congestion of mixed traffic (cars and buses) has been endogenously taken into account.

Different items in the individual commuting costs such as parking fees, bus fare, etc. have been computed in the model.

The population is located according to a minimization of the objective function (total costs).

The number of storeys at each location is obtained as a result of the efficient distribution of the population considering the irreversibility and durability characteristics explicitly.

The city structure both in plan and elevation is obtained as a result. A plan of the city including heights of buildings is obtained as a result.

Instead of conventional analytical methods, numerical procedures have been used in the model, methods such as Monte Carlo simulations, numerical optimization techniques, etc. Therefore, a digital electronic computer was necessary in order to carry out all the computational work.

Further research

The research on the model presented in this paper continues. The aim is to relax some of the limiting assumptions used in order to take a step towards a more realistic simulation of the economics of city building. In particular the following aspects will be considered in this extended model:

The choice of residential heating system (district heating, electric heating, etc.). The simulation of a heating system presents many similarities to that of the transportation system.

The demand for habitable space is price and income dependent.

The city area is divided into several residential zones and these are ranked according to differences in environmental benefits. Different income classes exist. Using these assumptions segregation patterns can be simulated.

Different, separated centres of working places may occur. Thus the working population can be distributed to different, separated, city areas.

Of course, the relaxation of some of the assumptions will not be presented in a definite manner. For instance, the problem of several, separate working place centres has been dealt with in a very simplified although usual way. The mechanism used to allocate workers among different working place centres has been some type of gravity rule without a more thorough economic foundation. In fact the solution of this problem is strongly dependent on the possibilities to simulate and evaluate the agglomeration effects in cities.

Another important aspect in city planning, that as far as we know has not yet been modelled, concerns the actual differences in tastes existing among the inhabitants. Such differences in taste can be expressed as differences in the demand for habitable space per person, time value, choice of mode of commuting and heating, etc. The distribution of such differences in taste among the inhabitants can be simulated by means of probability theory.

Although some efforts are undertaken in order to make the model more suitable as a practical tool for planning, it has not yet been applied to real city situations. Therefore it is a natural further step in this research to adapt the new extended model under work to the problems of a particular real city. Also when using this extended model, it is necessary to be aware of its remaining limiting assumptions.

Even with such a generalized model it is the authors' opinion that it can only be a partial tool in planning of city building.

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